THE ADVANCEMENT OF GRAPH THEORY IN OPERATION RESEARCH

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Abstract:-

Graph theory is a very natural and powerful tool in operation research. Graph theory is applied to operation research problems.Locating the shortest path between two vertices in a graph,obtaining an optimal matching of jobs and men,finding the shortest spanning tree in a weighted graph,The travelling salesman problem etc.are some examples of the use of graph theory in operation research.Here some more problems in OR using graph theoretic tools are taken.The related areas of OR in which graph theory is used most frequently are Transport network ,Activity network and theory of games.

Keywords:-Graph,Weighted graph,digraph,Transport network,Game theory.

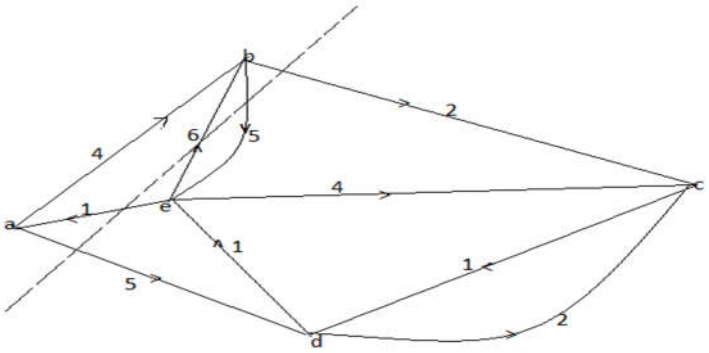
Introduction:-

The general problem of a Transport network is to maximize the flow or minimize the cost of a prescribed flow.One of the most popular and successful applications of network in OR is in the planning and scheduling of large complicated projects by activity network.The graph theoretic approach has been found to be computationally more efficient.The theory of games has became an important field of mathematical research since the publication of the first book on the subject by John von Neumann &Oskar Morgenstern in 1944.Game theory is applied to problems in engineering,economics,and war science to find the optimal way of performing certain tasks in a competitive environment.

1 **Transport Network**

A simple, connected,weighted,digraph G is called a transport network if the weight associated with every directed edge in G is a non negative number.In transport network this number represents the capacity of the edge and is written as c_{ij} for the edge directed from vertex i to vertex j .

Here(fig 1.1) a transport network is shown,here the numbers written besides the edge are the edge capacities.The capacities c_{ij} of an edge (i,j) is the maximal amount of some commodity(such as electrical energy,gas, water ,number of cars etc.) that can be transported from i to j ,along edge (i,j) per unit of time in a steady state.

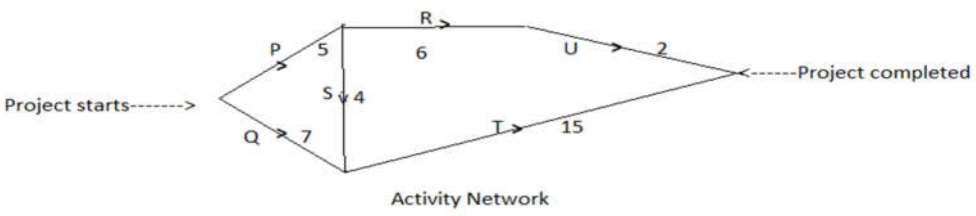


Transport network Fig(1.1)

2 Activity Network-

The most popular application of networks in operation research is in the planning and scheduling of large complicated projects.eg-CPM(Critical Path Method) and PERT(Program Evaluation and Review Technique).The project is divided into many well defined and non overlapping individual jobs,called activities.Each activity rquires a certain time called the duration of the activity The list of activities in a project,The list of immediate predessors for each activity,and the durations,a weighted graph can be drawn to depict the project,here each edge represents an activity and its weight represents the duration of the activity.The vertices represents beginning and ending of activities and are called events in the project.An activity cannot be started before all activities leading to the event have been completed.Each event in the project is a well defined occurrence in time.Such a weighted, connected digraph representing activities in a project is called activity network.

Eg-Suppose we have a project containing six activities P,Q,R,S,T and U with the restriction that P must precede Rand S;Qand S must precede T,R must precede U.The durations for the activities P,Q,R,S,Tand U are 5 ,7 ,6,4,15and 2 days respectively.It is observed that



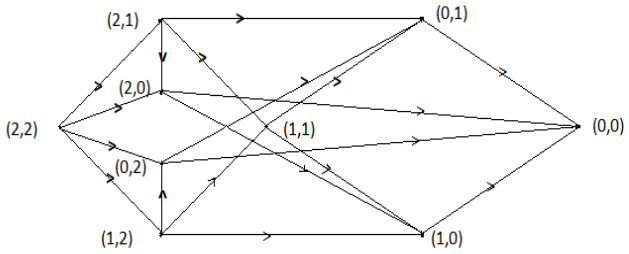
Fig(2.1)

It is observed that …

An activity network must be acyclic.The vertex denoting the start of the project must have zero in-degree.The vertex denoting the termination of the project must have zero-out degree,as no activity follows this vertex.

3.Game Theory-

The general idea of game theory is the same such as checkers,bridge and chess.Agame may be played between two persons or among more more than two persons,The former is called a two-person game and the latter as n-person game.A game is called finite if each player has a finite number of moves.An infinite game is one in which a player chooses a move from an infinite set of moves. To study a two person ,perfect information finite games without chance moves,A diagraph is a natural representation for such game.The vertices represents the positions and the edges represents the moves of the game.Their is a directed edge from vertex $v_i$tov_j if and only if the game can be transformed from position $v_i$tov_j by a move permissible as per rules of the game.Eg-Simplified Nim fig(3.1) (Aclassic problem in operation research and game theory,an ancient mathematical game of strategy)



Simplified Nim game

Fig(3.1)

Two piles of sticks are given and players P andQ take their turns,each taking any number of sticks from any one pile.The player who takes the last stick wins and the finite quantity of stick will eventually be exhausted,the ame has no draw.In fig(3.1) the complete game is described by the digraph.Here each state of game is described by an ordered pair (a,b) indicating the number of sticks in the first and second pile.

From the diagraph we observe that.

*It has a unique vertex with a zero in- degree which represents the starting position of the game.Vertex(2,2) is the starting vertex game.Vertex(2,2) is the starting vertex.

*It has one or more vertices with zero out- degree which corresponds to the closing position of the game.Vertex (0,0) is the closing vertex.

*It is connected ,acyclic digraph and each directed path from starting vertex to closing vertex represents one complete play of the game.

In the real problems in operation research the theory of games provides an approach rather than a complete analysis.Moreover graph theory is applicable only to a special and important class of games.

CONCLUSION-

The problems which we have considered in combinatorial operation research transportation problems,activity networks and game theory.They can be expressed as graph theory problems which involves connected and weighted digraphs.Graph theory has numerous applications in the above three such as route optimization,traffic flow ,modeling network design,scheduling, project management,resource allocation,network formation games,cooperative games,non cooperative games etc. Many more problems of OR (Assignment problem,Travelling sales man problem,Modelling,Minimum cost path,scheduling problem …)can be solved by the use of graph theory. graph theory is a fundamental tool in OR enabling us to tackle complex problems and optimize systems networks and processes.

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