Nanotopology and its applications

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Abstract

In this paper, we investigate the difference and relationship among Multi-Granular nano topology and Multi*-Granular nano topology based on their approximations and also by means of calculating the significant measure named as nano degree of dependence of decision attribute for the problem [6].

Key words

Multi Granular Nanotopology, Nanotopology, Nano measure, Nano approximations

1. INTRODUCTION

In (1982), Pawlak proposed the theory of rough sets for handling ambiguity, imprecision, and uncertainty in data analysis. He proposed the lower and upper approximations for handling the vagueness (ambiguity) are originating from the rough set, based on the equivalence relation that generated from the data. According to this approach, a concept can be classified as having ambiguous data if it has two or more definite sets (namely, its lower and upper approximations). The main ideas in the approach of rough sets are represented by these approximations, which are clearly described in terms of equivalence classes. The approximations can be used to derive new information about the concept. For instance, in decision rules (or decision-making), the upper approximation describes objects that could potentially belong to the concept while the lower approximation shows the characteristics of objects that are definitely related to the concept. As a result, the primary tool to determine the accuracy or vagueness of the set is the difference between the upper and lower approximation (which represents the boundary region). Hence, by determining whether the region of the boundary is empty or not, we may determine the accuracy or roughness of any subset. If a subset's boundary region is not empty, it can't be described because there isn't enough information. These are the main reasons why the idea of rough sets has achieved so much in various domains. The fundamental approach of Pawlak's philosophy, however, is the definition of approximations using a partition produced by an equivalence relation. Because it only deals with an information system with complete information, this relation has created a barrier to the field of application [5].

Lellis Thivagar et al [1, 3] interjected a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The nano-open sets are the components of a nano topological space. Yet, some nano words are sufficient to simply denote "extremely small." It derives from the Greek word "Nanos," which in contemporary science refers to a "dwarf" or a billionth of something. An example is the nanocar. The topology that is suggested here is called such due to its size, as it only has a maximum of five elements.

In the granular computing point of view nano topological space is based on single granulation, its named as the indiscernibility relation. This model of a nano topological space has recently been expanded to include a multi-granular nano topological space based on multiple equivalence relations, in which the set approximations are determined by employing multiple equivalence relations on the universe. Also, a fundamental properties of nano topological space has been made, along with a number of basic features [2].

In this paper, we investigate the difference and relationship among Multi-Granular nano topology and Multi*-Granular nano topology based on their approximations and also by means of calculating the significant measure named as nano degree of dependence of decision attribute for the problem [6].

2.PRELIMINARIES

Definition 2.1: [4]Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \{\cup\{R(x): R(x) \subseteq X\}\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \{\cup\{R(x): R(x) \cap X \neq \emptyset\}\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by

$$B_R(X) = U_R(X) - L_R(X).$$

Definition 2.2: [4]Let U be the universe, R be an equivalence relation on U and

$\tau_R(X) = \{ U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$.
(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii)The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ forms a topology on called as the nano topology on U with respect to We call, (U , $\tau_R(X)$) as the nano topological space.

Definition 2.3: [4] Let U be the universe and P, Q be any two equivalence relations on U and

$\tau_{P+Q}(X) = \{ U, \emptyset, L_{P+Q}(X), U_{P+Q}(X), B_{P+Q}(X)\}$ where $X \subseteq U$.

(i) $L_{P+Q}(X) = \cup\{[x]: P(x) \subseteq X or\ Q(x) \subseteq X\}$.
(ii) $U_{P+Q}(X) = \cup\{[x]: P(x) \cap X \neq \emptyset\ and\ Q(x) \cap X \neq \emptyset \}$.

(iii) $B_{P+Q}(X) = U_{P+Q}(X) - L_{P+Q}(X)$.

That is, $\tau_{P+Q}(X)$ forms a topology on U called as the Multi-granular nano topology on U with respect to X. We call (U , $\tau_{P+Q}(X)$) as the Multi-granular nano topological space.

Definition 2.4: [4] Let U be the universe and P, Q be any two equivalence relations on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. Let $X \subseteq U$.

(i) The Multi*- lower approximation of X with respect to P and Q is the set of all objects, which can be for certain classified as X with respect to P and Q and it is denoted by $L_{P*Q}(X)$. That is, $L_{P*Q}(X) = \bigcup_{x\in U}\{[x]: P(x) \subseteq X\ and\ Q(x) \subseteq X\}$, where P(x) and Q(x) denotes the equivalence class determined by x.
(ii) The Multi*- upper approximation of X with respect to $P\ and\ Q$ is the set of all objects, which can be possibly classified as X with respect to P and Q it is denoted by $U_{P*Q}(X)$. That is, $U_{P*Q}(X) = \bigcup_{x\in U}\{[x]: P(x) \cap X \neq \emptyset\ or\ Q(x) \cap X \neq \emptyset \}$. Where P(x) and Q(x) denotes the equivalence class determined by x.
(iii) The Multi* -boundary region of X with respect to P and Q is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_{P*Q}(X)$. That is,
$$B_{P*Q}(X) = U_{P*Q}(X) - L_{P*Q}(X).$$

Definition 2.5: [4]Let U be the universe and P, Q be any two equivalence relations on U and

$\tau_{P*Q}(X) = \{ U, \emptyset, L_{P*Q}(X), U_{P*Q}(X), B_{P*Q}(X)\}$ where $X \subseteq U$.

(i) $L_{P*Q}(X) = \cup\{[x]: P(x) \subseteq X\ and\ Q(x) \subseteq X\}$.
(ii) $U_{P*Q}(X) = \cup\{[x]: P(x) \cap X \neq \emptyset\ or\ Q(x) \cap X \neq \emptyset \}$.

(iii) $B_{P*Q}(X) = U_{P*Q}(X) - L_{P*Q}(X)$.That is, $\tau_{P*Q}(X)$ forms a topology on U called as the Multi*-granular nano topology on U with respect to X. We call (U , $\tau_{P*Q}(X)$) as the Multi*-granular nano topological space.

Example 2.6: Let U = {a, b, c, d} and R(x) be the equivalence relation on U. Let X = {a, c} $\subseteq U$. Then $L_R(X)$ = {a}, $U_R(X)$ = { a, b, c } and $B_R(X)$ = {b, c}, hence the nano topology $\tau_R(X)$ = { U, $\emptyset$,{a}, { a, b, c },{b, c} }.

Example 2.7. Let U = {a,b,c,d,e} and U/P = {{a},{b,c,d},{e}} and U/Q = {{b},{a},{c,d,e}} be two equivalence relations on U and let X = {a,c,d} $\subseteq U$. Then $L_{P+Q}(X)$ = {a},$U_{P+Q}(X)$ = {a} and $B_{P+Q}(X) = \emptyset$, hence the Multi -granular nano topology $\tau_{P+Q}(X)$ = { U, $\emptyset$,{a} }.

Example 2.8: Let U = {a,b,c,d,e} and U/P = {{a},{b,c,d},{e}} and U/Q = {{b},{a},{c,d,e}} be two equivalence relations on U and let X = {a,c,d} $\subseteq U$. Then $L_{P*Q}(X)$ = {a},$U_{P*Q}(X)$ = {a,d,e} and $B_{P*Q}(X)$ = {d,e}, hence the Multi* -granular nano topology $\tau_{P*Q}(X)$ = { U, $\emptyset$,{a},{a,d,e},{d,e} }.

3. PROPORTION BASED ON MEASURES

In this section, we have defined a new measure nano accuracy in terms of knowledge granulation and investigated the difference and relationship among MGNT and M*GNT based on their approximations and also by means of calculating their nano accuracy and degree of dependence of decision attribute to finding the key section for students to enhance their successful career.

Definition 3.1: Let (U ,A) be an information system where U is an non-empty finite set of objects, A is a finite set of attributes and A is divided into a set C of conditional attributes and a set D of decision attributes.

Definition 3.2. Let (U ,A) be an information system, let (U , $\tau_R(X)$) be a nano topological space and X $\subseteq U$ then the nano accuracy of X is defined as $NA_R(X)$= $1 - \xi_R(X)GK(R)$, where $\xi_R(X)$ = $1-\frac{|L_R(X)|}{|U_R(X)|}$ and GK(R)= $\frac{1}{|U|}[X_1^2+ X_2^2+\ldots+X_i^2]$.

Definition 3.3: Let (U , $\tau_{P+Q}(X)$) be a Multi-granular nano topological space and X $\subseteq U$, then the multi nano accuracy of X is defined as $NA_{P+Q}(X)$= $1 - \xi_{P+Q}(X)GK(P+Q)$, where $\xi_{P+Q}(X)$ = $1-\frac{|L_{P+Q}(X)|}{|U_{P+Q}(X)|}$ and GK(P+Q)= $GK(P) + GK(Q)$.

Definition 3.4. Let (U , $\tau_{P*Q}(X)$) be a Multi*-granular nano topological space and X $\subseteq U$, then the multi*- nano accuracy of X is defined as $NA_{P*Q}(X)$= $1 - \xi_{P*Q}(X)GK(P*Q)$, where $\xi_{P*Q}(X)$ = $1-\frac{|L_{P*Q}(X)|}{|U_{P*Q}(X)|}$ and GK(P*Q)= $GK(P) * GK(Q)$.

Definition 3.5. Let (U , $\tau_R(X)$) be a nano topological space and let $X \subseteq U$. Let U /D = $\{D_1, D_2,\ldots D_k\}$ be all decision classes induced by decision attribute D and A is divided into a set C of condition attributes, then the nano degree of dependence is defined as γ[P, D] = $\frac{1}{|U|}$[|$L_P(D_1)$| +| $L_P(D_2)$|].

4.ANALYSIS AND COMPARISON

Now, we can consider the problem of finding the difference and relationship based on their approximations and also by finding the nano accuracy and nano degree dependence in all the cases, to finding the key section for students to enhance their successful career [6]. College life is useful for the growth of students. For this growth the library plays a vital role. The library is of great help in the fulfilment of their goals, ambitions and inclinations, as it provides ample opportunities for acquiring knowledge and directing them to the road of a successful career. If the students use the library properly they will be benefited in getting a reputed job during or after their college life. To make the students prepare for different professions and occupations, and to develop their skills, library procures General Knowledge books, Competitive examination books, Newspapers etc. Consider the following table in which the information about the reading habits of 15 students are given.

A set valued ordered information system is presented in the above table, where $U = \{S_1, S_2, S_3, \cdots, S_{15}\}$ and $A = \{a_1, a_2, a_3, a_4, d\}$, $a_1 =$ Group I, $a_2 =$ Group II, $a_3 =$ Group III, $a_4 =$ Group IV of the reading habit groups and d is the decision as to whether the student is successful or not. The attribute set A is divided into two classes- a class C of condition attributes namely a_1, a_2, a_3, a_4 and d of decision attribute. The set of attribute values is given by $V = \{S,\ C,\ G,\ N,\ M,\ Y,\ P,\ B,\ T,\ O,\ n,\ e\}$ where $S, C, G, N, M, Y, P, B, T, O, n, and\ e$ respectively stands for Subject books, Competitive examination books, General Knowledge books, Newspapers, Magazines, Yearbooks, Project, Back volume, Thesis, OPAC, net, e- journals.

Table 1- Information table for reading habits of 15 students

Students	Group I (a_1)	Group II (a_2)	Group III (a_3)	Group IV (a_4)	Decision
S_1	$\{S, C, G\}$	$\{N, M, Y\}$	$\{P, B\}$	$\{n, e\}$	Successful
S_2	$\{S, G\}$	$\{N\}$	$\{P\}$	$\{O, n\}$	Unsuccessful
S_3	$\{S, C\}$	$\{N, M, Y\}$	$\{P, T\}$	$\{n, e\}$	Successful
S_4	$\{S, C, G\}$	$\{N, M\}$	$\{B, T\}$	$\{n\}$	Successful
S_5	$\{S, C, G\}$	$\{N, Y\}$	$\{P, B, T\}$	$\{O, n, e\}$	Successful
S_6	$\{S\}$	$\{N\}$	$\{P, B\}$	$\{n\}$	Unsuccessful
S_7	$\{S\}$	$\{N\}$	$\{P, B, T\}$	$\{n, e\}$	Unsuccessful
S_8	$\{S, C\}$	$\{N, M, Y\}$	$\{P, B, T\}$	$\{O, n, e\}$	Successful
S_9	$\{S\}$	$\{N, M, Y\}$	$\{B\}$	$\{O, n\}$	Unsuccessful
S_{10}	$\{C, G\}$	$\{N, M, Y\}$	$\{P, B, T\}$	$\{O, n, e\}$	Successful
S_{11}	$\{S\}$	$\{N\}$	$\{P\}$	$\{O\}$	Unsuccessful
S_{12}	$\{S\}$	$\{N\}$	$\{P, B, T\}$	$\{O, n, e\}$	Unsuccessful
S_{13}	$\{S, C, G\}$	$\{N, M\}$	$\{P, T\}$	$\{n, e\}$	Successful
S_{14}	$\{S, C, G\}$	$\{N, M, Y\}$	$\{P\}$	$\{O, n\}$	Successful
S_{15}	$\{S, G\}$	$\{M\}$	$\{P, B, T\}$	$\{O, n, e\}$	Unsuccessful

The domains are as follows:

V_{a_1} = {Subject books, Competitive examination books, General Knowledge books}

V_{a_2} = {Newspapers, Magazines, Yearbooks}

V_{a_3} = {Project, Back volume, Thesis}

V_{a_4} = {OPAC, net, e- journals}From the table, we can find that

U/a_1={{ $S_1, S_4, S_5, S_{13}, S_{14}$},{$S_2, S_{15}$},{$S_3, S_8$},{$S_6, S_7, S_9, S_{11}, S_{12}$},{$S_{10}$}}
U/a_2={{$S_1, S_3, S_8, S_9, S_{10}, S_{14}$},{$S_2, S_6, S_7, S_{11}, S_{12}$},{$S_4, S_{13}$},{$S_5$},{$S_{15}$}}
U/a_3={{S_1, S_6},{S_2, S_{11}, S_{14}},{S_3, S_{13}},{S_4},{$S_5, S_7, S_8, S_{10}, S_{12}, S_{15}$},{$S_9$}}
U/a_4={{S_1, S_3, S_7, S_{13}},{S_2, S_9, S_{14}},{S_4, S_6},{$S_5, S_8, S_{10}, S_{12}, S_{15}$},{$S_{11}$}

Case 1 Let us take X = {$S_2, S_{10}, S_{11}, S_{14}, S_{15}$}$\subseteq U$, then

$L_{a_1+a_3}$(X) = {$S_2, S_{15}, S_{10}, S_{11}, S_{14}$},$U_{a_1+a_3}$(X) = {$S_2, S_7, S_{10}, S_{11}, S_{12}, S_{15}$}

$L_{a_1*a_3}$(X) = {S_2},$U_{a_1*a_3}$(X) ={S_2 , $S_{15}, S_6, S_7, S_9, S_{11}, S_{12}, S_{14}, S_5, S_8, S_{10}$}

$\xi_{a_1+a_3}(X) = 1-\frac{|L_{a_1+a_3}(X)|}{|U_{a_1+a_3}(X)|} = 0.1667$, $\xi_{a_1*a_3}(X) = 1-\frac{|L_{a_1*a_3}(X)|}{|U_{a_1*a_3}(X)|} = 0.9091$

GK($a_1 + a_3$) = $GK(a_1) + GK(a_3)$
= 0.59 + 0.55
= 1.14

GK($a_1 * a_3$) = $GK(a_1) * GK(a_3)$
= 0.59 * 0.55
= 0.32

$NA_{a_1+a_3}(X) = 1 - \xi_{a_1+a_3}(X) GK(a_1 + a_3)$
= 0.809

$NA_{a_1*a_3}(X) = 1 - \xi_{a_1*a_3}(X) GK(a_1 * a_3)$
= 0.709

Hence by comparing the above measures we can conclude that

$NA_{a_1+a_3}(X) \geq NA_{a_1*a_3}(X)$

From the table, we have

$U/$ D = {D_S, D_U},

D_S = {$S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}$} and

D_U = {$S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}$}

Where $U/$ D = {{$S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}$},{$S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}$}}

From the table

L_{a_1}(D_S)= {$S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}$},$L_{a_1}$(D_U)= {$S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}$}

$\gamma[a_1, \text{D}] = \frac{1}{|U|}[\ |L_{a_1}(D_S)| + |L_{a_1}(D_U)|]$

$= \frac{1}{15}$ (15) = 1 L_{a_3}(D_S)= {S_3, S_4, S_{13}} L_{a_3}(D_U)= {S_9}

$\gamma[a_3, \text{D}] = \frac{1}{|U|}[\ |L_{a_3}(D_S)| + |L_{a_3}(D_U)|]$

$= \frac{1}{15}$ (4) $= \frac{4}{15}$

$L_{a_1+a_3}(D_S)$ = {$S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}$}

$L_{a_1+a_3}(D_U)$ = {$S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}$}

$L_{a_1*a_3}a_1$= {S_3, S_4, S_{13}}

$L_{a_1*a_3}(D_U)$ ={S_9}

$\gamma[a_1 + a_3, \text{D}] = \frac{1}{|U|}[\ |L_{a_1+a_3}(D_S)| + |L_{a_1+a_3}(D_U)|]$

$= \frac{1}{15}$ (15) = 1

$\gamma[a_1 * a_3, \text{D}] = \frac{1}{|U|}[\ |L_{a_1*a_3}(D_S)| + |L_{a_1*a_3}(D_U)|]$

$= \frac{1}{15}$ (4) $= \frac{4}{15}$

Hence, from the above it can be found that,
$\gamma[a_1 * a_3, \mathrm{D}] \leq \gamma[a_1, \mathrm{D}] \leq \gamma[a_1 + a_3, \mathrm{D}]$
$\gamma[a_1 * a_3, \mathrm{D}] \leq \gamma[a_3, \mathrm{D}] \leq \gamma[a_1 + a_3, \mathrm{D}]$

Case 2:

Let us take X = $\{S_2, S_9, S_{10}, S_{14}, S_{15}\} \subseteq U$, then
$L_{a_1+a_4}(\mathrm{X}) = \{S_2, S_{15}, S_{10}, S_9, S_{14}\}$
$U_{a_1+a_4}(\mathrm{X}) = \{S_2, S_5, S_9, S_{10}, S_{12}, S_{14}, S_{15}\}$
$L_{a_1*a_4}(\mathrm{X}) = \{S_2\}$, $U_{a_1*a_4}(\mathrm{X}) = \{S_1, S_4, S_5, S_{13}, S_{14}, S_2, S_{15}, S_6, S_7, S_9, S_{11}, S_{12}, S_{10}\}$
$\xi_{a_1+a_4}(X) = 1 - \frac{|L_{a_1+a_4}(X)|}{|U_{a_1+a_4}(X)|} = 0.2857$, $\xi_{a_1*a_4}(X) = 1 - \frac{|L_{a_1*a_4}(X)|}{|U_{a_1*a_4}(X)|} = 0.9231$
$\mathrm{GK}(a_1 + a_4) = GK(a_1) + GK(a_4)$
$= 0.59 + 0.55$
$= 1.14$
$\mathrm{GK}(a_1 * a_4) = GK(a_1) * GK(a_4)$
$= 0.59 * 0.55$
$= 0.32$

$NA_{a_1+a_4}(X) = 1 - \xi_{a_1+a_4}(X) GK(a_1 + a_4)$
$= 0.6743$
$NA_{a_1*a_4}(X) = 1 - \xi_{a_1*a_4}(X) GK(a_1 * a_4)$
$= 0.7049$
Hence by comparing the above measures we can conclude that
$NA_{a_1+a_4}(X) \leq NA_{a_1*a_4}(X)$
From the table, we have
$U/\mathrm{D} = \{D_S, D_U\}$,
$D_S = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$ and
$D_U = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
Where $U/\mathrm{D} = \{\{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}, \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}\}$
From the table
$L_{a_1}(D_S) = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$
$L_{a_1}(D_U) = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
$\gamma[a_1, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_1}(D_S)| + |L_{a_1}(D_U)|\,]$
$= \frac{1}{15}(15) = 1$
$L_{a_4}(D_S) = \emptyset \quad L_{a_4}(D_U) = \{S_{11}\}$
$\gamma[a_4, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_4}(D_S)| + |L_{a_4}(D_U)|\,]$
$= \frac{1}{15}(1) = \frac{1}{15}$

$L_{a_1+a_4}(D_S) = \{S_1, S_4, S_5, S_{13}, S_{14}, S_3, S_8, S_{10}\}$
$L_{a_1+a_4}(D_U) = \{S_2, S_{15}, S_6, S_7, S_9, S_{11}, S_{12}\}$
$L_{a_1*a_4}(D_S) = \emptyset$
$L_{a_1*a_4}(D_U) = \{S_{11}\}$
$\gamma[a_1 + a_4, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_1+a_4}(D_S)| + |L_{a_1+a_4}(D_U)|\,]$
$= \frac{1}{15}(15) = 1$
$\gamma[a_1 * a_4, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_1*a_4}(D_S)| + |L_{a_1*a_4}(D_U)|\,]$

$= \frac{1}{15}$ (1) $= \frac{1}{15}$

Hence, from the above it can be found that,
$\gamma[a_1 * a_4, \mathrm{D}] \leq \gamma[a_1, \mathrm{D}] \leq \gamma[a_1 + a_4, \mathrm{D}]$
$\gamma[a_1 * a_4, \mathrm{D}] \leq \gamma[a_4, \mathrm{D}] \leq \gamma[a_1 + a_4, \mathrm{D}]$

Case 3:
Let us take X = $\{S_1, S_2, S_3, S_8, S_{15}\} \subseteq U$, then $L_{a_1+a_2}(\mathrm{X}) = \{S_1, S_2, S_3, S_8, S_{15}\}$
$U_{a_1+a_2}(\mathrm{X}) = \{S_1, S_2, S_3, S_5, S_8, S_{10}, S_{14}, S_{15}\}$ $L_{a_1*a_2}(\mathrm{X}) = \{S_{15}\}$
$U_{a_1*a_2}(\mathrm{X}) = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}\}$
$\xi_{a_1+a_2}(X) = 1 - \frac{|L_{a_1+a_2}(X)|}{|U_{a_1+a_2}(X)|} = 0.375$
$\xi_{a_1*a_2}(X) = 1 - \frac{|L_{a_1*a_2}(X)|}{|U_{a_1*a_2}(X)|} = 0.9333$
GK$(a_1 + a_2) = GK(a_1) + GK(a_2)$
$= 0.59 + 0.67$
$= 1.26$
GK$(a_1 * a_2) = GK(a_1) * GK(a_2)$
$= 0.59 * 0.67$
$= 0.3953$

$NA_{a_1+a_2}(X) = 1 - \xi_{a_1+a_2}(X) GK(a_1 + a_2)$
$= 0.5275$
$NA_{a_1*a_2}(X) = 1 - \xi_{a_1*a_2}(X) GK(a_1 * a_2)$
$= 0.6311$
Hence by comparing the above measures we can conclude that
$NA_{a_1+a_2}(X) \leq NA_{a_1*a_2}(X)$
From the table, we have
$U/\mathrm{D} = \{D_S, D_U\}$,
$D_S = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$ and
$D_U = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
Where $U/\mathrm{D} = \{\{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}, \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}\}$
From the table
$L_{a_1}(D_S) = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$ $L_{a_1}(D_U) = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
$\gamma[a_1, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_1}(D_S)| + |L_{a_1}(D_U)|\,]$
$= \frac{1}{15}$ (15) = 1
$L_{a_2}(D_S) = \{S_4, S_{13}, S_5\}$
$L_{a_2}(D_U) = \{S_2, S_6, S_7, S_{11}, S_{12}, S_{15}\}$
$\gamma[a_2, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_2}(D_S)| + |L_{a_2}(D_U)|\,]$
$= \frac{1}{15}$ (9) $= \frac{3}{5}$

$L_{a_1+a_2}(D_S) = \{S_1, S_4, S_5, S_{13}, S_{14}, S_3, S_8, S_{10}\}$
$L_{a_1+a_2}(D_U) = \{S_2, S_{15}, S_6, S_7, S_9, S_{11}, S_{12}\}$
$L_{a_1*a_2}(D_S) = \{S_4, S_{13}, S_5\}$
$L_{a_1*a_2}(D_U) = \{S_2, S_6, S_7, S_{11}, S_{12}, S_{15}\}$

$\gamma[a_1 + a_2, \mathrm{D}] = \frac{1}{|U|}[\ |L_{a_1+a_2}(D_S)| + |\ L_{a_1+a_2}(D_U)|]$
$= \frac{1}{15}$ (15) = 1
$\gamma[a_1 * a_2, \mathrm{D}] = \frac{1}{|U|}[\ |L_{a_1*a_2}(D_S)| + |\ L_{a_1*a_2}(D_U)|]$
$= \frac{1}{15}$ (9) $= \frac{3}{5}$

Hence, from the above it can be found that,
$\gamma[a_1 * a_2, \mathrm{D}] \leq \gamma[a_1, \mathrm{D}] \leq \gamma[a_1 + a_2, \mathrm{D}]$
$\gamma[a_1 * a_2, \mathrm{D}] \leq \gamma[a_2, \mathrm{D}] \leq \gamma[a_1 + a_2, \mathrm{D}]$

Case 4:
Let us take X = $\{S_1, S_3, S_8, S_9, S_{10}, S_{14}\} \subseteq U$, then
$L_{a_2+a_3}(\mathrm{X}) = \{S_1, S_3, S_8, S_9, S_{10}, S_{14}\}$
$U_{a_2+a_3}(\mathrm{X}) = \{S_1, S_3, S_8, S_9, S_{10}, S_{14}, S_{15}\}$
$L_{a_2*a_3}(\mathrm{X}) = \{S_9\}$
$U_{a_2*a_3}(\mathrm{X}) = \{S_1, S_3, S_8, S_9, S_{10}, S_{14}, S_{15}, S_6, S_2, S_{11}, S_{13}, S_5, S_7, S_{12}\}$
$\xi_{a_2+a_3}(X) = 1 - \frac{|\ L_{a_2+a_3}(X)|}{|\ U_{a_2+a_3}(X)|} = 0.1429$
$\xi_{a_2*a_3}(X) = 1 - \frac{|\ L_{a_2*a_3}(X)|}{|\ U_{a_2*a_3}(X)|} = 0.9286$
GK$(a_2 + a_3) = GK(a_2) + GK(a_3)$
= 0.67 + 0.55
= 1.22
GK$(a_2 * a_3) = GK(a_2) * GK(a_3)$
= 0.67 * 0.55
= 0.3685

$NA_{a_2+a_3}(X) = 1 - \xi_{a_2+a_3}(X)GK(a_2 + a_3)$
= 0.8527
$NA_{a_2*a_3}(X) = 1 - \xi_{a_2*a_3}(X)GK(a_2 * a_3)$
= 0.6579
Hence by comparing the above measures we can conclude that
$NA_{a_2+a_3}(X) \geq NA_{a_2*a_3}(X)$
From the table, we have
$U/\mathrm{D} = \{D_S, D_U\}$,
$D_S = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$ and
$D_U = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
Where $U/\mathrm{D} = \{\{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\},\{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}\}$
From the table
$L_{a_2}(D_S) = \{S_4, S_{13}, S_5\}$
$L_{a_2}(D_U) = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
$\gamma[a_2, \mathrm{D}] = \frac{1}{|U|}[\ |L_{a_2}(D_S)| + |\ L_{a_2}(D_U)|]$
$= \frac{1}{15}$ (9) $= \frac{3}{5}$

$L_{a_3}(D_S) = \{S_3, S_4, S_{13}\}$
$L_{a_3}(D_U) = \{S_9\}$

$\gamma[a_3, \mathrm{D}] = \frac{1}{|U|}[\ |L_{a_3}(D_S)| + |\ L_{a_3}(D_U)|]$
$= \frac{1}{15}\ (\ 4\ ) = \frac{4}{15}$

$L_{a_2+a_3}(D_S\ ) = \{S_4, S_{13}, S_5, S_3\}$
$L_{a_2+a_3}(D_U\ ) = \{S_2, S_6, S_7, S_{11}, S_{12}, S_{15}, S_9\}$
$L_{a_2*a_3}(D_S) = \{\ S_4, S_{13}\}$
$L_{a_2*a_3}(D_U\ ) = \emptyset$
$\gamma[a_2 + a_3, \mathrm{D}] = \frac{1}{|U|}[\ |L_{a_2+a_3}(D_S)| + |\ L_{a_2+a_3}(D_U)|]$
$= \frac{1}{15}\ (\ 11\ ) = \frac{11}{15}$
$\gamma[a_2 * a_3, \mathrm{D}] = \frac{1}{|U|}[\ |L_{a_2*a_3}(D_S)| + |\ L_{a_2*a_3}(D_U)|]$
$= \frac{1}{15}\ (\ 2\ ) = \frac{2}{15}$

Hence, from the above it can be found that,
$\gamma[a_2 * a_3, \mathrm{D}] \leq \gamma[a_2, \mathrm{D}] \leq \gamma[a_2 + a_3, \mathrm{D}]$
$\gamma[a_2 * a_3, \mathrm{D}] \leq \gamma[a_3, \mathrm{D}] \leq \gamma[a_2 + a_3, \mathrm{D}]$

Case 5:
Let us take X = $\{S_1, S_5, S_8, S_{10}, S_{12}, S_{15}\} \subseteq U$, then
$L_{a_2+a_4}(\ \mathrm{X}\ ) = \{S_1, S_5, S_8, S_{10}, S_{12}, S_{15}\}$
$U_{a_2+a_4}(\ \mathrm{X}\ ) = \{S_1, S_3, S_5, S_8, S_{10}, S_{12}, S_{15}\}$
$L_{a_2*a_4}(\ \mathrm{X}\ ) = \{S_5\}$
$U_{a_2*a_4}(\ \mathrm{X}\ ) = \{S_1, S_3, S_8, S_9, S_{10}, S_{14}, S_2, S_6, S_7, S_{11}, S_{12}, S_5, S_{15}\}$
$\xi_{a_2+a_4}(X) = 1 - \frac{|\ L_{a_2+a_4}(\ X\ )|}{|\ U_{a_2+a_4}(\ X\ )|} = 0.1429$
$\xi_{a_2*a_4}(X) = 1 - \frac{|\ L_{a_2*a_4}(\ X\ )|}{|\ U_{a_2*a_4}(\ X\ )|} = 0.9231$
GK$(a_2 + a_4) = GK(a_2) + GK(a_4)$
= 0.67 + 0.55
= 1.22
GK$(a_2 * a_4) = GK(a_2) * GK(a_4)$
= 0.67 * 0.55
= 0.3685
$NA_{a_2+a_4}(X) = 1 - \xi_{a_2+a_4}(X) GK(a_2 + a_4)$
= 0.8257
$NA_{a_2*a_4}(X) = 1 - \xi_{a_2*a_4}(X) GK(a_2 * a_4)$
= 0.6598
Hence by comparing the above measures we can conclude that
$NA_{a_2+a_4}(X) \geq NA_{a_2*a_4}(X)$
From the table, we have
$U/\ \mathrm{D} = \{D_S, D_U\}$,
$D_S = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$ and
$D_U = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
Where $U/\ \mathrm{D} = \{\{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}, \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}\}$
From the table
$L_{a_2}(\ D_S) = \{S_4, S_{13}, S_5\}$

$L_{a_2}(D_U)= \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
$\gamma[a_2, \text{D}] = \frac{1}{|U|}[\, |L_{a_2}(D_S)| + |L_{a_2}(D_U)|]$
$= \frac{1}{15}(9) = \frac{3}{5}$

$L_{a_4}(D_S)= \emptyset$
$L_{a_4}(D_U)= \{S_{11}\}$
$\gamma[a_4, \text{D}] = \frac{1}{|U|}[\, |L_{a_4}(D_S)| + |L_{a_4}(D_U)|]$
$= \frac{1}{15}(1) = \frac{1}{15}$

$L_{a_2+a_4}(D_S) = \{S_4, S_{13}, S_5, S_3\}$
$L_{a_2+a_4}(D_U) = \{S_2, S_6, S_7, S_{11}, S_{12}, S_{15}, S_9\}$
$L_{a_2*a_4}(D_S) = \emptyset$
$L_{a_2*a_4}(D_U) = \{S_{11}\}$
$\gamma[a_2 + a_4, \text{D}] = \frac{1}{|U|}[\, |L_{a_2+a_4}(D_S)| + |L_{a_2+a_4}(D_U)|]$
$= \frac{1}{15}(11) = \frac{11}{15}$
$\gamma[a_2 * a_4, \text{D}] = \frac{1}{|U|}[\, |L_{a_2*a_4}(D_S)| + |L_{a_2*a_4}(D_U)|]$
$= \frac{1}{15}(1) = \frac{1}{15}$

Hence, from the above it can be found that,
$\gamma[a_2 * a_4, \text{D}] \leq \gamma[a_2, \text{D}] \leq \gamma[a_2 + a_4, \text{D}]$
$\gamma[a_2 * a_4, \text{D}] \leq \gamma[a_4, \text{D}] \leq \gamma[a_2 + a_4, \text{D}]$

Case 6:
Let us take X = $\{S_1, S_2, S_9, S_{11}, S_{14}, S_{15}\} \subseteq U$, then
$L_{a_3+a_4}(\text{X}) = \{S_1, S_2, S_9, S_{11}, S_{14}, S_{15}\}$
$U_{a_3+a_4}(\text{X}) = \{S_1, S_2, S_5, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{14}, S_{15}\}$
$L_{a_3*a_4}(\text{X}) = \{S_2, S_9, S_{11}, S_{14}\}$
$U_{a_3*a_4}(\text{X}) = \{S_1, S_6, S_2, S_{11}, S_{14}, S_5, S_8, S_{10}, S_{12}, S_{15}, S_9, S_3, S_7, S_4\}$
$\xi_{a_3+a_4}(X) = 1 - \frac{|L_{a_3+a_4}(X)|}{|U_{a_3+a_4}(X)|} = 0.4545$
$\xi_{a_3*a_4}(X) = 1 - \frac{|L_{a_3*a_4}(X)|}{|U_{a_3*a_4}(X)|} = 0.7143$
$\text{GK}(a_3 + a_4) = GK(a_3) + GK(a_4)$
$= 0.55 + 0.55$
$= 1.1$
$\text{GK}(a_3 * a_4) = GK(a_3) * GK(a_4)$
$= 0.55 * 0.55$
$= 0.3025$
$NA_{a_3+a_4}(X) = 1 - \xi_{a_3+a_4}(X) GK(a_3 + a_4)$
$= 0.49995$
$NA_{a_3*a_4}(X) = 1 - \xi_{a_3*a_4}(X) GK(a_3 * a_4)$
$= 0.7839$
Hence by comparing the above measures we can conclude that
$NA_{a_3+a_4}(X) \leq NA_{a_3*a_4}(X)$

From the table, we have
$U/$ D = $\{D_S, D_U\}$,
$D_S = \{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}$ and
$D_U = \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}$
Where $U/$ D = $\{\{S_1, S_3, S_4, S_5, S_8, S_{10}, S_{13}, S_{14}\}, \{S_2, S_6, S_7, S_9, S_{11}, S_{12}, S_{15}\}\}$
From the table
$L_{a_3}(D_S) = \{S_3, S_4, S_{13}\}$
$L_{a_3}(D_U) = \{S_9\}$
$\gamma[a_3, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_3}(D_S)| + |L_{a_3}(D_U)|\,]$
$= \frac{1}{15}(4) = \frac{4}{15}$

$L_{a_4}(D_S) = \emptyset$
$L_{a_4}(D_U) = \{S_{11}\}$
$\gamma[a_4, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_4}(D_S)| + |L_{a_4}(D_U)|\,]$
$= \frac{1}{15}(1) = \frac{1}{15}$

$L_{a_3+a_4}(D_S) = \{S_3, S_4, S_{13}\}$
$L_{a_3+a_4}(D_U) = \{S_9, S_{11}\}$
$L_{a_3*a_4}(D_S) = \emptyset$
$L_{a_3*a_4}(D_U) = \emptyset$
$\gamma[a_3 + a_4, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_3+a_4}(D_S)| + |L_{a_3+a_4}(D_U)|\,]$
$= \frac{1}{15}(5) = \frac{1}{3}$
$\gamma[a_3 * a_4, \mathrm{D}] = \frac{1}{|U|}[\,|L_{a_3*a_4}(D_S)| + |L_{a_3*a_4}(D_U)|\,]$
$= \frac{1}{15}(\emptyset) = \emptyset$

Hence, from the above it can be found that,
$\gamma[a_3 * a_4, \mathrm{D}] \leq \gamma[a_3, \mathrm{D}] \leq \gamma[a_3 + a_4, \mathrm{D}]$
$\gamma[a_3 * a_4, \mathrm{D}] \leq \gamma[a_4, \mathrm{D}] \leq \gamma[a_3 + a_4, \mathrm{D}]$

5. CONCLUSION

The Nano accuracy of *X* with respect to all the conditional attributes of the Multi-granular nano topology increases than Multi*-granular nano topology and hence the Multi-granular nano topology becomes finer than the Multi*-granular nano topology. From the contribution, it can be found that when two attribute sets in library section possesses a contradiction or inconsistent relationship.

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